NORMAL FORM, SET CLASS, AND PRIME FORM

"...we seek to discover the relationships that underlie
the surface and lend unity and coherence to musical works."

Joseph Straus, *Introduction to Post-Tonal Theory*

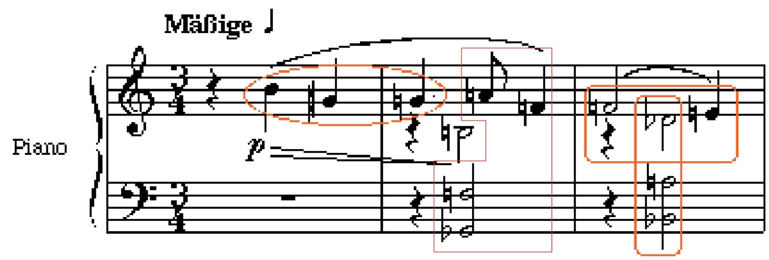
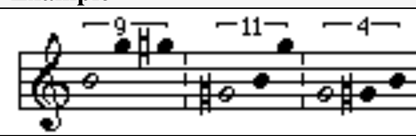
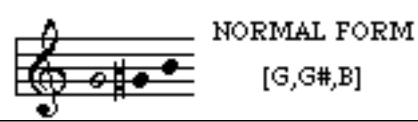


Fig. 1. Schoenberg, Piano Piece, op. 11, no. 1, mm. 1-3.

Normal Form

The normal form is the most compact way of notating a pc set. Use the following 4-step procedure to find the **normal form**.[1]

Procedure to Determine the Normal Form of a PC Set

Step	Description	Example
1.	List the **ascending orderings** of the pc set. The number of ascending orderings is equal to the number of members in the set.	
2.	The ascending ordering with the **least outside interval** is the normal form.	
3.	If a tie results from the previous step, compare the **next-most-outside intervals**, the interval between the first and second-to-last notes, first-and-third to last notes, etc. Select the ascending ordering with the least outside interval.	
4.	If there is still a tie after applying repeated applications of the previous step, select the ordering that begins with the least first pc integer.	
	When only one ordering remains, you have found the **normal form** of the set. Square brackets are used to indicate the normal form of a set.	

[1] See Joseph Straus, *Introduction to Post-Tonal Theory* (Upper Saddle River, NJ: Prentice Hall, 2005), 35-38. Straus's normal form procedure is based on the algorithm given in John Rahn, *Basic Atonal Theory* (New York: Longman, 1980). It should be mentioned that this algorithm is slightly different than the one given in Allen Forte, *The Structure of Atonal Music* (New Haven, CT: Yale University Press, 1973).

Prime Form

Use the following 4-step procedure to find the **prime form** of a pc set.[2]

Procedure to Determine the Prime Form of a PC Set

Step	Description	Example
1.	Determine the normal form of the set.	Normal form is: [7,8,11]
2.	Determine which T_n operation will transpose the set so that its first member is 0, and apply it to the set.	T_5 [7,8,11] = **[0,1,4]**
3.	Invert the set and repeat the previous two steps for the inverted set.	[7,8,11] inverted is: (5,4,1) 3.1 Normal form is: [1,4,5] 3.2 T_{11} [1,4,5] = **[0,3,4]**
4.	Compare the normal forms produced by Steps 2 and 3. Select the form that is most compact to the left as the prime form.	Compare [0,1,4] and [0,3,4] Prime form is: **(014)**

Shortcut Method for Prime Form[3]

Step	Description	Staff Notation
1.	Determine the normal form of the set.	
2.	*Checking the T_n forms* As shown in the diagram (right), label the first pc of the normal form 0 and calculate the intervallic distance *up* to each member of the set in *left-to-right* order using semitones. Record each distance below the set.	
3.	*Checking the T_nI forms* As shown in the diagram, label the last pc of the normal form 0 and calculate the intervallic distance *down* to each member of the set in *right-to-left* order using semitones. Record each distance above the set.	
4.	Compare the normal forms produced by Steps 2 and 3. Select the form that is most compact to the left as the prime form.	Compare: (014) and (034) **PRIME FORM** **(014)**

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[2] See Joseph Straus, *Introduction to Post-Tonal Theory* (Upper Saddle River, NJ: Prentice Hall, 2005), 35-38.
[3] The short-cut method does not work for all pc sets. The method exploits the fact that, for many pc sets, the interval pattern of the retrograde of the normal form is equivalent to the interval pattern for the inversion of the normal form.

Set Class

A set class is a group of pc sets related by T_n and T_nI. There are usually 24 distinct members of a set class–12 under T_n, and 12 under T_nI. **Symmetical sets**, sets that map onto themselves under T_n (at levels other than T_0) or T_nI, have fewer than 24 **distinct forms**.

The 12 trichordal set classes produced under T_n and T_nI are listed below.

Forte name	**Prime form**
3-1	(012)
3-2	(013)
3-3	(014)
3-4	(015)
3-5	(016)
3-6	(024)
3-7	(025)
3-8	(026)
3-9	(027)
3-10	(036)
3-11	(037)
3-12	(048)

Set classes are usually referred to by their Forte name, or Forte name followed by their prime form, for example, set class 3-3 (014). The *Set Class List* appendix lists all set classes of cardinality 3 through 9 inclusive.

Set Class 3-3 (014)			
T_0	[C,C#,E]	T_0I	[G#,B,C]
T_1	[C#,D,F]	T_1I	[A,C,C#]
T_2	[D,D#,F#]	T_2I	[A#,C#,D]
T_3	[D#,E,G]	T_3I	[B,D,D#]
T_4	[E,F,G#]	T_4I	[C,D#,E]
T_5	[F,F#,A]	T_5I	[C#,E,F]
T_6	[F#,G,A#]	T_6I	[D,F,F#]
T_7	[G,G#,B]	T_7I	[D#,F#,G]
T_8	[G#,A,C]	T_8I	[E,G,G#]
T_9	[A,A#,C#]	T_9I	[F,G#,A]
T_{10}	[A#,B,D]	$T_{10}I$	[F#,A,A#]
T_{11}	[B,C,D#]	$T_{11}I$	[G,A#,B]

Fig. 2. The 24 members of set class 3-3 (014).

Set Notation Summary

	Letter name	*Integer*
PC set:	(B,G#,G)	(11,8,7)
Normal form:	[G,G#,B]	[7,8,11]
Prime form:	(014)	
Forte name:	3-3	
Set class:	3-3 (014)	