

The Number of Intervals Formed by a PC Set

The *cardinality* (c) of a set refers to the number of distinct elements in the set. The number of intervals formed (n) by a set is directly related its cardinality. The relation can be expressed by the following formula:

$$n = \frac{c^2 - c}{2}$$

c	n	Number of intervals formed
1	0	0
2	1	0 + 1
3	3	0 + 1 + 2
4	6	0 + 1 + 2 + 3 + 4
5	10	0 + 1 + 2 + 3 + 4 + 5
6	15	0 + 1 + 2 + 3 + 4 + 5 + 6
7	21	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7
8	28	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8
9	36	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
10	45	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
11	55	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11
12	66	0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12

For example, the set (C,C#,D) contains the intervals:

$$C-C\#, C-D, \text{ and } C\#-D$$

The set (Bb,A,C,B) contains the intervals:

$$Bb-A, Bb-C, Bb-B, A-C, A-B, \text{ and } C-B$$

Source

Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973), p. 19.